Ph.D. COMMON ENTRANCE TEST AUGUST 2024
SUBJECT – MATHEMATICS

PART B

Roll No:

Duration: 60 minutes **Maximum Marks: 50**

Instructions:

1. **This entrance test question paper is not to be taken out of the examination hall**
2. **Question paper consists of Section A and Section B**
3. **Section A consists of 30 MCQs carrying 1 Mark each. Write the Alphabet of the correct answer in the space given.**
4. **Section B consists of Descriptive questions carrying 5 marks each. Restrict your answer to 500 words. Additional plain sheets have been attached to the question paper to answer Section B**

SECTION – A

Answer the following questions by writing the Alphabet of the correct answer in the Box given: **30 X 1 = 30**

1.	The rank of the matrix: $A=\begin{pmatrix}1&2&3&2\\2&3&5&1\\1&3&4&5\end{pmatrix}$ is A. 1 B. 2 C. 3 D. 4
2	The one value of λ for which the following system 3x-y+ λz=1,2x+y+z=2,x+2y- λz=-1 will fail to have unique solution is A. λ=7/2 B. λ=3/2 C. λ=-3/2 D. λ=-7/2
3.	Eigen value of the matrix $\begin{pmatrix}1&2\\3&2\end{pmatrix}$ is A. 4, 1 B. -1,4

	C. 3,2 D. 5,3
4	The solution set of $x_1 > 2$ & $x_2 < 4$ is the convex set. A. (1,2) B. [0,4) C. [2,4] D. (2,4)
5	One bag contains 3 white and 2 black balls, another bag contains 5 white and 3 black balls. If a bag is chosen at random and a ball is drawn from it, what is the chance that it is white? A. 3/8 B. 49/80 C. 8/13 D. 1/80
6.	If a fair coin is tossed 10 times, then the probability of obtaining atleast one head is A. 1/1024 B. 17/1024 C. 1023/1024 D. 1
7.	The general solution of $\frac{dy}{dx} = 1 + x^2 + y^2 + x^2y^2$ is A. $\tan^{-1}x = y + (y^3/3) + C$ B. $\tan^{-1}x = x + (x^3/3) + C$ C. $\tan^{-1}x = \tan^{-1}y + C$ D. $\tan^{-1}x + \tan^{-1}y = C$
8.	The solution of $y\frac{dy}{dx} - \frac{d^2y}{dx^2} = 2x$ is A. $y = x^2+2x+2$ B. $y = x^3+2x^2+x$ C. $y = 2x+2$ D. $y = 3x^2+5x+2$
9	Which of the following differential equation represents the family of circles touching the x-axis at the origin? A. $(x^2-y^2)\,dy - 2xy\,dx = 0$ B. $(x^2+y^2)\,dy + 2xy\,dx = 0$ C. $(x^2-y^2)\,dy + 2xy\,dx =$ D. $(x^2+y^2)\,dy - 2xy\,dx = 0$
10.	The solution of the differential equation $dy=(1+y^2)\,dx$ is A. $y = \tan x + C$ B. $y = \tan (x + C)$ C. $x = \tan^{-1}(y+ C)$ D. $2x = \tan^{-1}(y+ C)$
11.	The solution of the differential equation $\ln\left(\frac{dy}{dx}\right) - a = 0$ is A. $y = xe^a+C$ B. $x = ye^a+C$

	C. y = ln x +C D. x = ln y + C
12.	Reduced form of Cauchy's homogeneous differential equation in to standard linear differential equation with constant coefficients is $x^3\frac{d^3y}{dx^3}+2x\frac{dy}{dy}-2y=x^2\log x+3x$ A. $\left(D_1^3-3D_1^2+4D_1-2\right)y=e^{2z}z+3e^z$ B. $\left(D_1^3-3D_1^2+4D_1-2\right)y=0$ C. $\left(D_1^3-3D_1^2+4D_1-2\right)x=e^{2z}z+3e^z$ D. $\left(D_1^3-3D_1^2+4D_1-2\right)x=e^{2z}z-3e^z$
13.	In the equation $\frac{d^2y}{dx^2}+P\frac{dy}{dx}+Qy=R$ if $1+P+Q=0$ then CF is A. e^{-x} B. e^x C. x D. x^2
14.	The complimentary function for the differential equation $x\frac{d^2y}{dx^2}-2(x+1)\frac{dy}{dx}+(x+2)y=(x-2)e^{2x}:(x>0)$ is A. e^{-x} B. x C. e^x D. x^2
15	The PDE formed from $2z=\frac{x^2}{a^2}+\frac{y^2}{b^2}$ where $p=\frac{x}{a^2}$ and $q=\frac{y}{b^2}$ A. xp-yq=2z B. xp+yq=2z C. xq+yp=2 D. xq-yp=z

16	One solution of the PDE $p\tan x+q\tan y=\tan z$ is A. $\frac{\sin x}{\sin y}=c_1$ B. $\sin x-\sin y=0$ C. $\sin x-\sin y=c_2$ D. $\frac{\sin y}{\sin x}=c_1$
17.	$\Delta\log x$ =? A. $\log(x+h)$ B. $\log x-\log(x+h)$ C. $\log x+\log(x+h)$ D. $\log\left(1+\frac{h}{x}\right)$
18.	Forward difference operator $\Delta^n f(x)$ is A. $\frac{h^n a_n}{n!}$ B. $n!h^n a_n$ C. $nh^{n+1}a_n$ D. $n^2h^n a_n$
19.	Applying Power method on the matrix A=$\begin{bmatrix}4 & 2\\ 1 & 3\end{bmatrix}$ gives largest eigen value is A.3.41 B. 4.98 C. 3.14 D. 4.112
20.	Jacobi iteration on the system of equations: 5x-y+3z = 10 3x+6y = 18 x+y+5z = -10 taking 3,0 and -2 as initial values gives the answer. A. 4,1,3 B. 4,1,-3 C. 4,-1,3 D. -4,1,3

21	The root of x^3-x-2=0 near x=1.5 by Newton Rapshon gives the value A. 1.765 B. 1.521 C. 1.988 D. 1.342
22	$\Delta e^{2x} = ?$ A. $e^{2x}(1-e^{2h})$ B. $e^{2h}(1-e^{2x})$ C. $e^{2x}(e^{2h}-1)$ D. $e^{2x}(e^{2h}+1)$
23	If f(x)= 2^x, $\Delta f(x) = 2^x$ *and* $\Delta g(x) = x.x!$,g(x)=x! and h=1 then $\Delta \frac{f(x)}{g(x)}$ is A. $\frac{(1-x)2^x}{(x+1)!}$ B. $\frac{(1+x)2^x}{(x-1)!}$ C. $\frac{2^x}{(x-1)!}$ D. $\frac{2^x}{(x+1)!}$
24	If f(x)=(1-x)(1-2x)(1-3x)(1-4x), the value of $\Delta^4[f(x)]$ taking h=1 is A. 570 B. 576 C. 581 D. 587
25	If f(x)=x(x+2), the value of $\Delta f(x)$ taking h=1 is A. 2x+3 B. 2x-3 C. 3-2x

D. 5-2x

26. If $\Delta^3(1+\alpha x)(1-2x)(1+4x) = -144$ *for* $h = 1$ then 'α' is

A. 1

B. 2

C. 3

D. 4

27. The value of $\Delta^6\left[3x^6 + 4x^5 + 2x^4 - x^3 + 2x^2 - 3x + 1\right]$ *at* $h = 1$ is

A. 2100

B. 2170

C. 2110

D. 2160

28. A function is specified by the following table. Finding y^1 at x=1 by numerical differentiation we use

x	1.00	1.05	1.10	1.15	1.20	1.25	1.30
y	1.00	1.0247	1.0488	1.0723	1.0954	1.1180	1.1401

A. $\frac{dy}{dx} = \frac{1}{h}\left[\Delta y_0 - \frac{1}{2}\Delta^2 y_0 + \frac{1}{3}\Delta^3 y_0 - \frac{1}{4}\Delta^4 y_0 + ...\right]$

B. $\frac{dy}{dx} = \frac{1}{h}\left[\Delta y_0 + \frac{2p-1}{2}\Delta^2 y_0 + \frac{3p^2 - 6p + 2}{6}\Delta^3 y_0 + ...\right]$

C. $\frac{dy}{dx} = \frac{1}{h}\left[\nabla y_n + \frac{2p+1}{2}\nabla^2 y_n + \frac{3p^2 + 6p + 2}{6}\nabla^3 y_n + ...\right]$

D. $\frac{d^2y}{dx^2} = \frac{1}{h^2}\left[\Delta^2 y_0 - \Delta^3 y_0 + \frac{11}{12}\Delta^4 y_0 - .....\right]$

29. The method which is not used for numerical integration is

A. Weddle's rule.

B. Trapezoidal rule.

	C. Taylor's rule D. All of the above.
30.	$\int_{x_0}^{x_0+nh} f(x)dx = \frac{h}{2}\left[\left(y_0+y_n\right)+2\left(y_1+y_2+.....+y_{n-1}\right)\right]$ is formula is called A. Simpson's one-third formula. B. Simpson's three-eight formula. C. Trapezoidal rule formula. D. Weddle's formula

Section - B

Answer any four questions (Each question carry 5 marks 4*5 = 20

1. Verify the following system of equations for consistency and if consistent solve: x+y+z=4, 2x+y-z=1, x-y+2z=2.

2. Find the Expectation and Variance of the following probability distribution.

X	-3	-1	0	1	3
P(X)	0.05	0.45	0.20	0.25	0.05

3. Solve : $x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+9y=3x^2+Sin(3\log x)$.

4. Solve: PDE: $(D^2 - DD' + D'-1)\, z=e^{x+2y}$.

5. The manager of an oil refinery must decide on the optimal mix of two possible blending process of which the inputs and outputs per production run as follows:

process	Input units		Output(units)	
	Crud A	crude B	Gasoline X	Gasoline Y
1	5	3	5	8
2	4	5	4	4

The maximum amounts available of crude A and B are 200 units and 150 units respectively. Market requirements show that at least 100 units of gasoline X and 80 units of gasoline Y must be produced. The profits per production run from process 1 and process 2 are Rs.300 and Rs.400 respectively. Solve LPP by graphical method.

6. Find a root of the equation f(x)=0 given that f(30)=-30,f(34)=-13, f(38)=3,f(42)=18, by using Lagrange's interpolation formula.